19) $F(x) = \frac{1}{2}x^4 + 2x^3$ $(-2,-8)$ $(0,0)$

$F'(x) = 2x^3 + 6x^2$

$F''(x) = 6x^2 + 12x$

$0 = 6x^2 + 12x$

$0 = 6x(x+2)$

$0, -2$

$F''(-3) = 18$

$F''(-2) = 0$

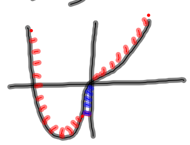
concave up: $(-\infty,-2) \cup (0,\infty)$

concave down: $(-2,0)$

$F''(-1) = -6$

$F''(0) = 0$

$F''(1) = 18$



23) $f(x) = \frac{1}{4}x^4 - 2x^2$

$f'(x) = x^3 - 4x$

$F''(x) = 3x^2 - 4$

$0 = 3x^2 - 4$

$4 = 3x^2$

$\frac{4}{3} = x^2$

$\pm\frac{2}{\sqrt{3}} = x$

$F''(-2) = 8$ concave↑

$F''(-\frac{2}{\sqrt{3}}) = 0$

$F''(0) = -4$ concave↓

$F''(\frac{2}{\sqrt{3}}) = 0$

$F''(2) = 8$ concave↑

concave up: $(-\infty,-\frac{2}{\sqrt{3}}) \cup (\frac{2}{\sqrt{3}},\infty)$

concave down: $(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$

25) $F(x) = x(x-4)^3$

$F'(x) = 1(x-4)^3 + x\cdot 3(x-4)^2(1)$

$F''(x) = 3(x-4)^2 + 3(x-4)^2 + 3x\cdot 2(x-4)$

$= 3(x-4)^2 + 3(x-4)^2 + 6x(x-4)$

$= 6(x-4)^2 + 6x(x-4)$

$= 6(x-4)[x-4+x]$

$= 6(x-4)(2x-4)$

$0 = 12(x-4)(x-2)$

$4, 2$

concave up: $(-\infty,2) \cup (4,\infty)$

concave down $(2,4)$

We really are only interested in what is positive and negative.

$F''(1) = +$

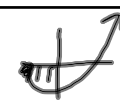
$F''(2) = 0$

$F''(3) = -$

$F''(4) = 0$

$F''(5) = +$

27) $x\sqrt{x+3} = F(x)$ $x \geq -3$



$F'(x) = \sqrt{x+3} + \frac{x}{2\sqrt{x+3}}$

$F''(x) = \frac{1}{2\sqrt{x+3}} + \frac{2\sqrt{x+3} - x\,\frac{1}{2\sqrt{x+3}}}{(2\sqrt{x+3})^2}$

$= \frac{1}{2\sqrt{x+3}} + \frac{4(x+3) - x}{2\sqrt{x+3}\cdot 4(x+3)}$

$= \frac{4(x+3) + 4(x+3) - x}{8\sqrt{x+3}\,(x+3)} = \frac{7x+24}{8\sqrt{(x+3)^3}}$

$x = \cancel{-\frac{24}{7}}, -3$

out of domain

Concave up $(-3,\infty)$

29) $F(x) = \frac{4}{x^2+1}$ Domain: $\mathbb{R}$

$F'(x) = \frac{0 - 4(2x)}{(x^2+1)^2} = \frac{-8x}{(x^2+1)^2}$ c.p. at $x=0$

$F''(x) = \frac{-8(x^2+1)^2 + 8x[2(x^2+1)\cdot 2x]}{(x^2+1)^4}$

$F''(1) = 2$ $= \frac{\cancel{(x^2+1)}[-8(x^2+1) + 32x^2]}{(x^2+1)^{\cancel{4}\,3}} = \frac{24x^2 - 8}{(x^2+1)^3} = \frac{8(3x^2-1)}{(x^2+1)^3}$

$F''(-\frac{1}{\sqrt{3}}) = 0$

$F''(0) = -8$

$F''(\frac{1}{\sqrt{3}}) = 0$ $F''(1) = 2$

$3x^2 - 1 = 0$ $x^2 = \frac{1}{3}$

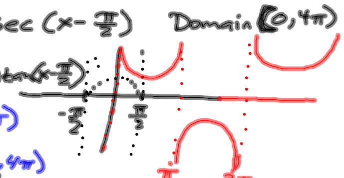
$3x^2 = 1$ $x = \pm\frac{1}{\sqrt{3}}$

concave up: $(-\infty,-\frac{1}{\sqrt{3}}) \cup (\frac{1}{\sqrt{3}},\infty)$

concave down: $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

33) $F(x) = \sec(x - \frac{\pi}{2})$ Domain $(0, 4\pi)$

$F'(x) = \sec(x-\frac{\pi}{2})\tan(x-\frac{\pi}{2})$



concave up $(0,\pi) \cup (2\pi,3\pi)$

concave down $(\pi,2\pi) \cup (3\pi,4\pi)$

$F''(x) = \sec(x-\frac{\pi}{2})\tan^2(x-\frac{\pi}{2}) + \sec^3(x-\frac{\pi}{2})$

undef: $0 + \pi n, n \in \mathbb{Z}$ → inflect. $\pi, 2\pi, 3\pi$

$0 = \sec(x-\frac{\pi}{2})[\tan^2(x-\frac{\pi}{2}) + \sec^2(x-\frac{\pi}{2})]$

$0 = \sec(x-\frac{\pi}{2})[\tan^2(x-\frac{\pi}{2}) + \tan^2(x-\frac{\pi}{2}) + 1]$

$0 = \sec(x-\frac{\pi}{2})[2\tan^2(x-\frac{\pi}{2}) + 1]$

$\sec(x-\frac{\pi}{2}) = 0$

secant can never equal zero.

$\varnothing$

$2\tan^2(x-\frac{\pi}{2}) = -1$

$\sqrt{\tan^2(x-\frac{\pi}{2})} = \sqrt{-\frac{1}{2}}$

Square root of a negative would be imaginary.

$\varnothing$